



### Ejercicio 44

**44** Definimos la *traza* de una matriz cuadrada  $A$  de orden 2 como  $tr(A) = a_{11} + a_{22}$ . Prueba que si  $A$  y  $B$  son dos matrices cuadradas de orden 2, entonces  $tr(A \cdot B) = tr(B \cdot A)$ .

#### Resolución

Si  $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$  y  $B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ , entonces:

$$A \cdot B = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix} \rightarrow tr(A \cdot B) = a_{11}b_{11} + a_{12}b_{21} + a_{21}b_{12} + a_{22}b_{22}$$

$$B \cdot A = \begin{pmatrix} b_{11}a_{11} + b_{12}a_{21} & b_{11}a_{12} + b_{12}a_{22} \\ b_{21}a_{11} + b_{22}a_{21} & b_{21}a_{12} + b_{22}a_{22} \end{pmatrix} \rightarrow tr(B \cdot A) = a_{11}b_{11} + a_{21}b_{12} + a_{12}b_{21} + a_{22}b_{22}$$

Por tanto,  $tr(A \cdot B) = tr(B \cdot A)$ .