17 Calcula la inversa de las siguientes matrices:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix}$$

Después, resuelve estas ecuaciones:

a) $AX = B$ **b)** $XB = A$

Resolución

- Calculamos $|A| = 3 - 2 = 1$

  Hallamos los adjuntos de los elementos de A:

$$A_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 3; \quad A_{12} = -\begin{vmatrix} 0 & 0 \\ 2 & 3 \end{vmatrix} = 0; \quad A_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 0 \end{vmatrix} = -2$$

$$A_{21} = -\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = -6; \quad A_{22} = \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 1; \quad A_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = 4$$

$$A_{31} = \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} = -1; \quad A_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0; \quad A_{33} = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1$$

$$Adj\,(A) = \begin{pmatrix} 3 & 0 & -2 \\ -6 & 1 & 4 \\ -1 & 0 & 1 \end{pmatrix} \rightarrow [Adj\,(A)]^t = \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix}$$

  Comprobación:

$$A \cdot A^{-1} = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $|B| = 2 + 6 - 6 = 2$

$$B_{11} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = -2; \quad B_{12} = -\begin{vmatrix} 0 & 3 \\ 2 & 1 \end{vmatrix} = 6; \quad B_{13} = \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = -2$$

$$B_{21} = -\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = -1; \quad B_{22} = \begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix} = 2; \quad B_{23} = -\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$$

$$B_{31} = \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} = 3; \quad B_{32} = -\begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = -6; \quad B_{33} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2$$

$$Adj\,(B) = \begin{pmatrix} -2 & 6 & -2 \\ -1 & 2 & 0 \\ 3 & -6 & 2 \end{pmatrix} \rightarrow [Adj\,(B)]^t = \begin{pmatrix} -2 & -1 & 3 \\ 6 & 2 & -6 \\ -2 & 0 & 2 \end{pmatrix}$$

$$B^{-1} = \frac{1}{2}\begin{pmatrix} -2 & -1 & 3 \\ 6 & 2 & -6 \\ -2 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -1/2 & 3/2 \\ 3 & 1 & -3 \\ -1 & 0 & 1 \end{pmatrix}$$

Comprobación:

$$B \cdot B^{-1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1/2 & 3/2 \\ 3 & 1 & -3 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

a) $AX = B \;\rightarrow\; A^{-1}AX = A^{-1}B \;\rightarrow\; X = A^{-1}B$

$$X = \begin{pmatrix} 3 & -6 & -1 \\ 0 & 1 & 0 \\ -2 & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -4 & -19 \\ 0 & 1 & 3 \\ -2 & 3 & 13 \end{pmatrix}$$

b) $XB = A \;\rightarrow\; XBB^{-1} = AB^{-1} \;\rightarrow\; X = AB^{-1}$

$$X = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} -1 & -1/2 & 3/2 \\ 3 & 1 & -3 \\ -1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3/2 & -7/2 \\ 3 & 1 & -3 \\ -5 & -1 & 6 \end{pmatrix}$$